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Solstability

Technical Reference

Limit equilibrium methods, slip surfaces and their search, water, loads, reinforcement and verification.

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1 Scope and notation

1.1 Purpose

This reference describes the engineering model and the numerical methods implemented in Solstability 1.0.1: the idealisation of the slope, the formation of slices, the limit equilibrium methods, the search for the critical slip surface, the treatment of water, loads, earthquake and reinforcement, the verification methodologies, and the checks of the implementation. It is intended to let an engineer judge whether the program is appropriate for a problem and to check its results independently. The use of the program is described in the User Manual (SPL-SS-002).

Limit equilibrium methods are well established in geotechnical practice, and the methods implemented here are those published in the technical literature cited at the end of this reference. Each method rests on assumptions about the forces between slices; for the same slip surface the methods can give somewhat different factors of safety. The choice of method, of the soil parameters, of the search ranges and of the required factor of safety remains the responsibility of the engineer.

1.2 Notation

Symbols are defined where they first appear. The principal symbols are listed below. Equations are written in consistent units (kN, m, kPa); forces are per metre of slope.

Table 1.1 — Principal symbols

Symbol	Meaning	Symbol	Meaning
b, l	width of a slice; length of its base	F	factor of safety
c, φ	cohesion and angle of shearing resistance (effective or total stress)	K_h, K_v	horizontal and vertical seismic coefficients
E, X	normal and shear interslice forces	R, x_c, y_c	radius and centre of a circular slip surface
N, N'	total and effective normal force on the base of a slice	r_u	pore pressure coefficient
T	mobilised shear force on the base	u, U	pore pressure; its resultant on the base
W	weight of a slice	α	inclination of the base of a slice
Q	surcharge resultant on a slice	δ, θ	inclination of the interslice force; its scale in Spencer and Morgenstern-Price
γ, γ_{sat}, γ_w	unit weight above water, saturated unit weight, unit weight of water	$f(\xi)$	interslice force function

1.3 Coordinates, sign convention and units

The model is a vertical cross-section in the x–y plane, x to the right and y upward (elevation). The analysis is plane strain and all forces are per metre of slope length; elements with a spacing along the slope, such as anchors, nails and piles, enter with their force divided by the spacing. A slope may face to either side: the program determines the direction of sliding from the slip surface (the mass moves from the higher end of the slip surface towards the lower end) and mirrors the model internally when the mass moves to the right, so that all equations below are written for a mass that slides to the left. Angles are entered in degrees and are positive counter-clockwise unless stated otherwise. Input and output may be in SI or US units; the computation is in SI.

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2 The slope model

2.1 Geometry

The ground consists of soil layers of given thickness below the model top, and of interface polygons that describe the slope above the layers, fills, or soil removed from them. The program combines them into regions, each filled with one soil. The ground surface is the upper boundary of all regions. Rigid bodies are regions of very high strength: a slip surface base inside one takes $c' = 10\ 000$ kPa and $\varphi' = 89°$ (on the dividing planes between Sarma blocks $c' = 200$ kPa and $\varphi' = 45°$), and a rigid body is drained during rapid drawdown. Within 1 cm of its outline the neighbouring soil applies. Incompressible subsoil is a soil that a slip surface may not enter. The model extends between the abscissae x_{min} and x_{max}; a slip surface must start and end on the ground surface inside that range.

Construction stages repeat the analysis for a sequence of states of the same slope. A stage may add an embankment or remove soil by an earth cut, and may change water, loads, reinforcement and the analysis settings. Each stage is analysed independently with the geometry valid in that stage.

2.2 Slip surfaces

A **circular** slip surface is defined by its centre (x_c, y_c) and radius R; the part of the circle below the ground surface between its two intersections with the surface is the slip surface. A **polygonal** slip surface is a sequence of points from left to right; its first and last points lie on or above the ground surface. Between its ends a slip surface may not rise above the ground surface.

2.3 Slices

The sliding mass is the soil between the ground surface and the slip surface. For a circular surface the horizontal range between the two ends is divided into 20 slices of equal width; for a polygonal surface the vertices of the polygon are the boundaries of the slices. Each slice is described by its width b, the inclination α and length $l = b/\cos\alpha$ of its base (the chord), its weight, the shear strength and pore pressure on its base, the loads on its top and the forces of reinforcement that it contains (Figure 2.1).

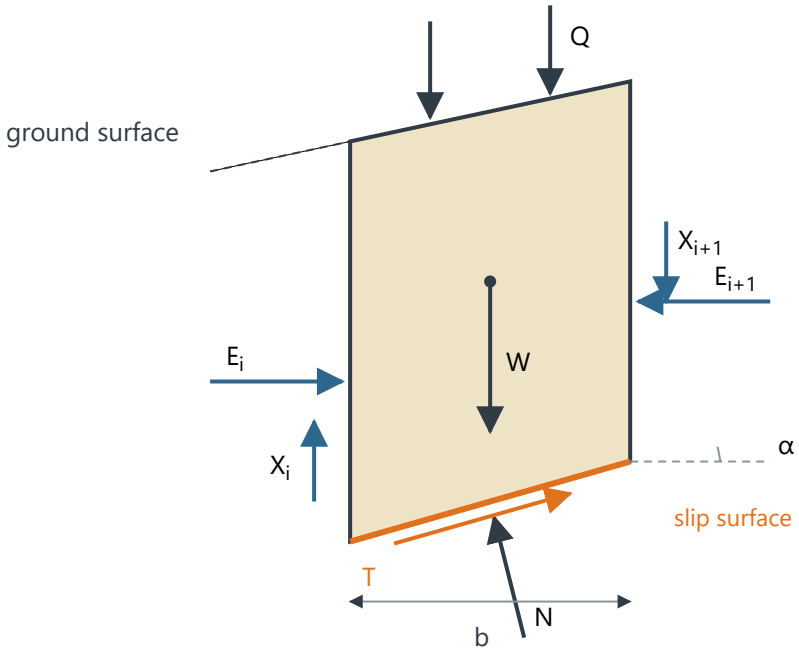


Figure 2.1 — Forces on a slice

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The **weight** of a slice is integrated over narrow vertical columns: the slice is divided into columns about 1/200 of the width of the slip surface wide, and along the vertical line through the middle of each column the length in each soil region is multiplied by the unit weight γ above the saturation line and γ_{sat} below it, and by the column width. The saturation line is the ground water table, or for rapid drawdown the original water table. The weight acts at the centroid found from the same columns.

The **shear strength** on the base follows the Mohr–Coulomb criterion

$$s = c + (\sigma_n - u) \tan \varphi \tag{2.1}$$

with c and φ of the soil at the base. Where the base of a slice crosses several soils, the base is divided into short sub-segments (the total length of the slip surface divided by 200) and the strength is integrated over them. A soil with **total stress** state uses its undrained shear strength c_u with $\varphi = 0$, and the pore pressure on its base is not taken into account. A soil with an **anisotropic** strength (foliation) uses a second pair of parameters c, φ where the base is inclined within the range of angles of the foliation.

2.4 Pore water pressure

The pore pressure on the base of each slice is integrated over the same sub-segments as the strength. The types of water input are the following.

Table 2.1 — Types of pore pressure input

Type	Pore pressure at a point (x, y) of the slip surface
Ground water table	$u = \gamma_w (y_w - y) \cos^2\beta$ below the table, 0 above it; y_w is the elevation of the table at x and β its inclination (seepage parallel to the table).
Coefficient r_u	$u = r_u \sigma_v$, with σ_v the total vertical stress at the point; a soil may have its own r_u.
Rapid drawdown	The weight below the original water table is saturated. The pore pressure is $u = \gamma_w (h_0 - X \Delta h) \cos^2\beta$ (not less than zero), where h_0 is the depth below the original table (limited to the ground surface, not above the new table and not above the highest point of the new table), Δh the drop of the table at the point (not more than h_0), β the inclination of the higher of the two tables and X the coefficient of the soil: $X = 1$ for a permeable soil, whose pore pressure follows the new table, $X = 0$ for an impermeable soil, which keeps the original pressure. Where the new table is not lower than the original one, the pore pressure follows the new table.
Ground water table and suction surface	As for the ground water table below the table; between the table and the suction surface the pore pressure is negative and hydrostatic, $u = -\gamma_w (y - y_w)$; above the suction surface it is zero.
Pore pressure isolines	u interpolated linearly with elevation between the two isolines of given pressure that enclose the point.

A soil may take its pore pressure from its own **piezometric surface** instead of the project water: at a point in that soil $u = \gamma_w (y_p - y)$ below the surface (y_p its elevation above the point; the full vertical head, without the inclination correction used for the project ground water table), and the soil is saturated below it. A soil may also have a **pore pressure constant** u_c, added to the pore pressure at every point of the soil. A negative constant gives a negative pore pressure, which is not limited.

Water above the ground surface, for example a reservoir or a pond at the toe, acts as a pressure $\gamma_w h_w$ normal to the surface and enters each slice as an external force. On the vertical sides of the slices the water pressure is integrated from the base to the ground surface where the method uses it. In the Bishop and Fellenius methods the net horizontal water force on a slice acts at the middle of the water column above the middle of its base.

2.5 Loads

Surcharges are applied to the slices whose top they cover. A **strip** load of intensity q and a **trapezoidal** load varying linearly from q_1 to q_2 act on the ground surface or at a given elevation inside the soil; a **concentrated** load and a **line** load act at a point. A load inside the soil, or a concentrated load on a footing of finite width, is spread at 2 (vertical) : 1 (horizontal) to the slices below it. A load may be inclined; its horizontal component acts on the slices as a horizontal force. Loads are classified as permanent, variable or accidental, which matters for the partial factors

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of the verification (Chapter 6); each load receives one factor, chosen from the slice below its centre of gravity: the load is unfavourable when $\tan\varphi\cos(\alpha+\theta)-\sin(\alpha+\theta)<0$ (α = inclination of the slice base, θ = inclination of the load) and favourable otherwise.

2.6 Earthquake

Earthquake loading is pseudo-static. Each slice receives a horizontal inertia force $K_h\,W$ at its centroid, directed out of the slope, and a vertical force $K_v\,W$ (positive downward). The coefficients are entered directly or follow from a seismic standard: GB 50111-2006 and JTG B02-2013 multiply K_h and K_v by importance and composite coefficients, JTG B02 also by a distribution coefficient that increases from 1.0 below a height of 20 m to 1.6 at the crest; NB 35047-2015 distributes the coefficients over the height of the slope, measured from the lowest to the highest ground point above the slip surface (a block whose centre of gravity lies at or below the lowest point keeps the coefficients unchanged); GB 50330-2013 uses the horizontal and vertical coefficients of the site directly. EN 1998-5 coefficients can be derived from the design ground acceleration ($K_h = \alpha\,S\,/\,r$).

2.7 Tensile crack

A tensile crack of depth z_c below the ground surface carries no shear strength: in the zone of the crack $c=\varphi=0$ on the slip surface. When the crack is filled with water, the hydrostatic pressure $\gamma_w\,d$ (d the depth below the top of the slip surface) acts horizontally on the slices whose base lies in the crack zone, each over the vertical extent of its base, at the centroid of its pressure trapezoid. Their sum is $\tfrac{1}{2}\,\gamma_w\,z_c^{\,2}$, acting at one third of the crack depth above its bottom.

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3 Limit equilibrium methods

3.1 General form

The factor of safety F is the factor by which the shear strength would have to be divided to bring the sliding mass into limiting equilibrium along the slip surface. On the base of each slice the mobilised shear force is

$$T = \frac{c\,l + N' \tan\varphi}{F}, \qquad N' = N - U \tag{3.1}$$

and the unknowns are F, the base forces N and the interslice forces E (normal) and X (shear) with their points of application. The problem is statically indeterminate; each method closes it with an assumption about the interslice forces and satisfies some or all of the equations of equilibrium (Table 3.1).

Table 3.1 — Methods and the equilibrium they satisfy

Method	Slip surface	Equilibrium	Assumption on the interslice forces
Fellenius / Petterson	circular	moment about the centre	interslice forces neglected
Bishop (simplified)	circular	moment; vertical force per slice	horizontal ($X = 0$)
Spencer	circular, polygonal	forces and moment	parallel, constant inclination θ
Morgenstern-Price	circular, polygonal	forces and moment	$X = \lambda\, f(\xi)\, E$
Janbu	circular, polygonal	forces; moment per slice	position of the line of thrust
Shahunyants	circular, polygonal	forces	horizontal; F applied to the driving forces
ITFM (implicit, explicit)	circular, polygonal	forces	parallel to the base of the slice above
Sarma	polygonal	forces and moment of each block	blocks with inclined sides; strength mobilised on the sides

In all methods the external forces on a slice — surcharges, water on the surface and on the sides, earthquake, and the forces of reinforcement — are added to the weight in the direction in which they act. Their treatment for each kind of reinforcement is described in Chapter 5.

3.2 Fellenius / Petterson

The ordinary method of slices (Fellenius 1936) neglects the interslice forces and takes the normal force on the base from the weight of the slice alone. Moment equilibrium about the centre of the circle gives

$$F = \frac{\sum \left[c\,l + (W\cos\alpha - U)\tan\varphi\right]}{\sum W \sin\alpha} \tag{3.2}$$

without iteration. The method is conservative for frictional soils with high pore pressure and is mainly of historical and comparative interest.

3.3 Bishop

The simplified method of Bishop (1955) assumes horizontal interslice forces. Vertical equilibrium of each slice and moment equilibrium of the mass about the centre of the circle give

$$F = \frac{\sum \left[c\,b + (W - u\,b)\tan\varphi\right]/m_\alpha}{\sum W \sin\alpha}, \qquad m_\alpha = \cos\alpha + \frac{\sin\alpha\tan\varphi}{F} \tag{3.3}$$

which is solved iteratively for F. Surcharges and earthquake forces enter the vertical equilibrium and the moment about the centre with their lever arms. The method is accurate for circular slip surfaces; where m_α becomes small or negative at the toe of a steep surface the solution is not meaningful; the analysis then reports that the method did not converge for that slip surface.

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3.4 Spencer

Spencer (1967) assumes that the interslice forces are parallel, with the same inclination θ for all boundaries. For a trial θ, the force equilibrium of the slices, solved from one end of the mass to the other with zero interslice force at both ends, gives a factor of safety F_f; the moment equilibrium gives F_m. The solution is the pair (F, θ) for which both are satisfied. The program iterates on θ and, where the iteration does not converge from the first estimate, repeats it from other starting values.

3.5 Morgenstern-Price

Morgenstern and Price (1965) relate the shear to the normal interslice force by

$$X = \lambda f(\xi) E, \qquad f(\xi) = \left|\sin\left(\pi \xi^{a}\right)\right|^{b} \tag{3.4}$$

where ξ is the relative horizontal position of the boundary in the mass (0 at one end, 1 at the other) and λ a scale found from the equilibrium. The interslice function is one of seven shapes: constant (equivalent to Spencer), half-sine (a = 1, b = 2) and the clipped sine functions with (a, b) = (1, 0.5), (0.5, 2), (0.5, 0.5), (2, 2) and (2, 0.5). Force and moment equilibrium are both satisfied, as in Spencer's method. The selected function is tried first; if it does not converge, the non-constant functions are tried in the order listed until one converges, and the results name the function used.

Spencer's method starts from θ = 0 and, if it does not converge, restarts from θ = 0.3, 0.4, 0.5, −0.1 and −0.2. Janbu's method first tries four assumed lines of thrust at about one third of the slice height (constant and three smooth variations) and then starts from the solutions of the Morgenstern-Price and Spencer methods.

3.6 Janbu

In Janbu's generalised procedure of slices (Janbu 1973) the interslice normal force acts on a line of thrust of assumed position. Moment equilibrium of each slice about the midpoint of its base then gives the interslice shear, and force equilibrium gives F. The line of thrust is first placed at a fraction of the height of each boundary above the slip surface; where the solution does not converge with this assumption, it is started from the solutions of the Morgenstern-Price and Spencer methods. The method satisfies the force equilibrium of the mass and the moment equilibrium of each slice.

3.7 Shahunyants

The method of Shahunyants assumes horizontal interslice forces and applies the factor of safety to the forces that drive the mass rather than to the strength. Each force acting on a slice is projected on the direction of the base: components in the direction of sliding are active, components against it passive. With horizontal interslice forces the factor of safety follows in closed form, without iteration, from the sums of the resisting and active terms.

3.8 ITFM

The imbalance thrust force method (implicit transfer coefficient method) of the Chinese standard GB 50330-2013 assumes that the interslice force acts parallel to the base of the slice above it. In the **implicit** form the factor of safety divides the strength of the soil, and F is found from the force equilibrium with zero interslice force at the toe. In the **explicit** form the factor of safety multiplies the driving forces and is obtained directly; where the interslice force in the upper part of the mass would become tensile, that part is removed from the transfer and F is computed again. The method assumes small changes of inclination between adjacent segments of a polygonal slip surface; where adjacent segments differ by more than 10° the program warns that the factor of safety may be overestimated.

3.9 Sarma

The method of Sarma (1973, 1979) divides the sliding mass into blocks whose sides may be inclined, and mobilises the shear strength on the sides as well as on the bases. For a given F the equilibrium of the blocks gives the critical horizontal acceleration K_c that brings the mass into limiting equilibrium; the factor of safety is the value of F for

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which $K_c(F)$ equals the horizontal seismic coefficient of the analysis (zero without earthquake). The blocks are bounded by the vertices of the polygonal slip surface; the inclinations of the inner sides are optimised in steps of 10°, 3° and 1° as long as F decreases and the shear forces on the sides and bases remain positive. The weight of a block, the strength on its base and sides (averaged over the length of each side) and the water force on each side are integrated over the block. With a seismic standard that distributes the coefficients over the height of the slope, Sarma uses the coefficients without that distribution; the program says so with the results.

3.10 Convergence

The iterative methods stop when the change of F between iterations and the residual of the equilibrium are below fixed tolerances; the sequence of iterations is fixed, so that the same input always gives the same result. A method that does not converge, or whose base normal forces become unrealistic, reports no factor of safety for that slip surface, and the optimization or search passes over it. Where a method that satisfies moment equilibrium does not converge because of large geotextile forces, the geotextile forces are scaled by a factor between 0 and 1 found by halving the interval until it is narrower than 0.005, within each starting value of the method; the factor applied is reported with the results.

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4 Search for the critical slip surface

4.1 Overview

The factor of safety of a slope is the smallest factor of safety of all kinematically possible slip surfaces. The program evaluates one given surface (standard analysis) or searches for the most critical surface by optimization, by a grid of circles or by random generation of trial surfaces. A search finds the most critical of the surfaces it examines; it cannot prove that no other surface is more critical. Results of different searches, or of the same search from a different start, should be compared before a slope is judged.

All searches respect the restrictions entered by the user: a trial surface that crosses a restriction segment is not admissible. Surfaces that enter incompressible subsoil, that leave the model, or for which the method does not converge, are not admissible either. The circular search also skips surfaces whose base runs through a rigid body; the polygonal and Sarma searches evaluate such surfaces with the rigid-body strength, and a Sarma result whose dividing plane cuts a rigid body carries a warning that it can be overestimated. The searches are deterministic: the same input gives the same result.

4.2 Optimization of a circle

A circle is described by three points: its two ends on the ground surface and the lowest point of the arc midway between them. The two ends move along the surface and the inner point moves freely, which gives four degrees of freedom. Starting from the input circle, the optimization tries changes of the four coordinates along a fixed set of directions with a common step; an improvement is accepted at once and the trial of directions starts again. When no direction improves the factor of safety the step is reduced. The first step is one fifth of the width of the model; the search ends when the step is below 1/2000 of the width. A second pass restarts from the result with the full step, to escape a local minimum. The ends can be kept fixed individually.

4.3 Optimization of a polygon

A polygonal surface is optimised by moving its points: the inner points in both directions, the end points along the ground surface. Points can be fixed, or restricted to horizontal or to vertical movement. Adjacent points are kept at least 0.05 m apart. The step is reduced as for a circle until it falls below the tolerance. Optimization by the surplus sliding force (Section 6.3) replaces the factor of safety by the force when the factor of safety is below the design value.

4.4 Grid search

A grid of centres is placed around the centre of the input circle: n_x positions on each side along the grid x direction and n_z along the grid z direction, with the given steps; the grid can be rotated. At each centre n_R radii on each side of the input radius are evaluated. The total number of circles is $(2n_x+1)(2n_z+1)(2n_R+1)$. The most critical circle becomes the input circle, so that the grid can be refined around it by halving the steps.

4.5 Random search

Random search generates trial surfaces with the technique of Siegel (1975). Circles and irregular surfaces start at initiation points spaced evenly over the initiation range and are built of straight segments of the given length (the last one shorter) until they reach the ground surface, which must happen inside the termination range. A slope rising to the left is mirrored, so that the surfaces are always generated from the toe side towards the crest side. Directions are measured from the horizontal towards the termination range, negative downward.

- **First segment**: its direction is random between a counterclockwise and a clockwise limit and is biased towards the clockwise one. With both limits zero they are automatic: 5° below the inclination of the ground surface at the initiation point, and −45°.

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- **Circles**: each following segment turns by the same random deflection, so the segments are chords of a circle. The range of the deflection is chosen so that the circle can reach the termination range; a surface that ends short of it or turns past the vertical narrows the range for the next attempt. The circle through the chord points is analysed.
- **Irregular surfaces**: the direction of each following segment is random, up to 45° counterclockwise and up to a random angle between 0° and 45° clockwise of the preceding one, biased towards the clockwise side, and never steeper than vertical.
- **Sliding blocks**: one random point is taken in each box (a centre line and a height); the points form the base of the block. From the point nearest the toe the surface rises to the ground at the passive angle $45° - \varphi/2$ and from the point nearest the crest at the active angle $45° + \varphi/2$. φ is that of the soil below the nearest layer boundary above the start of a segment; the direction changes where a segment crosses a layer boundary.

A surface that ends short of or beyond the termination range, or reaches below the lowest elevation (the bottom of the model unless a lower limit of the depth is given), is rejected and generated again; the rejection narrows the direction limits of the following attempts from the same initiation point. Restrictions deflect the surfaces: a restriction marked *Up* raises the clockwise limit (as bedrock does), one marked *Down* lowers the counterclockwise limit. A surface that cannot be generated in 200 attempts, or that needs more than 100 points, is skipped. Without user ranges, the initiation range is the lower (toe) part of the ground surface and the termination range its upper (crest) part. The generator uses a fixed seed, so the same input generates the same surfaces.

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5 Reinforcement

Reinforcement elements enter the equilibrium as forces per metre of slope, the force of one element divided by its spacing along the slope. An element acts on the sliding mass only where it crosses the slip surface or, for an anchor, where its head lies on the sliding mass. Active forces (anchors, piles) are applied as known external forces; passive forces are part of the resistance: nail forces are divided by the factor of safety in the methods that satisfy force equilibrium, while geotextile forces enter these methods at their full value (scaled down only when the method does not converge otherwise) and form their own force group in the thrust methods.

5.1 Anchors

A prestressed anchor applies its force at the head, in the direction of the anchor into the soil. It acts when the head is on the sliding mass. With the option *anchors infinite* the anchor always acts and the required free length (the distance from the head to the slip surface) is reported; otherwise the anchor acts fully when the slip surface crosses its free length, partly when it crosses the root (reduced linearly along the root) and not at all when it misses the anchor.

In the Bishop and Fellenius methods the anchor force is a resisting action: its components enter the normal force on the base of the slice divided by the factor of safety, and its moment about the centre of the circle is added to the resisting moment. In the methods that satisfy force equilibrium it acts as a fixed external force on the slice. In the Bishop and Fellenius methods passive reinforcement (nails, geotextiles) is treated like the anchor force: its components enter the normal force divided by the factor of safety.

5.2 Soil nails

A nail resists with a force along its axis at the point where it crosses the slip surface:

$$F = \min\left(T_p\, x;\, R_t;\, R_f + T_p\, y\right) \tag{5.1}$$

where x is the length of the nail behind the slip surface, y the length in front of it up to the head, R_t the tensile strength of the bar (entered, or from its diameter and yield strength) and R_f the resistance of the head (zero for a free head). The pull-out resistance per metre T_p is entered, or computed as $\pi\, d\, q_\mathrm{s}$ from the bond of the nail or of the soil, or from the effective stress:

$$T_p = \pi\, d\, \left(K_\alpha\, \sigma_z \tan\varphi + c\right), \qquad K_\alpha = \frac{1 + (1 - \sin\varphi)}{2} \tag{5.2}$$

or according to HA 68/94 (Highways Agency 1994):

$$T_p = \pi\, d\, \left(\sigma_n \tan\varphi + c\right), \qquad \sigma_n = \frac{(1 + K_L)\,\sigma_z}{2}, \qquad K_L = \frac{1 + K_a}{2}, \qquad K_a = \frac{1 - \sin\varphi}{1 + \sin\varphi} \tag{5.3}$$

with σ_z the effective vertical stress from the weight of the soil above the point (submerged unit weight below the water table) and φ, c of the soil at the point. For a computed T_p the lengths x and y are divided into segments (20 over the whole nail) and T_p is evaluated at the middle of each segment.

In the Bishop and Fellenius methods on a circular slip surface the nail force acts at the point where the nail crosses the slip surface. In the other methods, and on a polygonal slip surface, it is distributed over the slices between the nail head and that point, in proportion to the length of nail in each slice, at the middle of that length.

5.3 Geotextiles

A geotextile resists with a force along its direction where it crosses the slip surface, limited by its tensile strength R_t. The pull-out resistance of the embedded length l outside the sliding mass is computed as $F = \sum 2\, C\, \sigma'\, \tan\varphi\, \Delta x$ over 100 equal horizontal steps Δx along the embedded part, with the effective vertical stress σ' from the weight of the

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soil above each point (submerged unit weight below the water table) and the angle of internal friction φ of the soil at the point, C the interaction coefficient; or from the anchorage length l_k needed to develop the full strength, $F = (l/l_k)\ R_t$; or from an entered pull-out resistance per metre, $F = T_p\ l$. Rupture and pull-out behind the slip surface are always checked; pull-out in front of the slip surface is checked for a geotextile with a free end.

The force acts along the geotextile by default. An **inclination factor** f between 0 and 1 turns it towards the tangent of the slip surface at the intersection: the angle of the force is $\theta = \theta_r + f\ (\theta_t - \theta_r)$, with θ_r the direction of the geotextile and θ_t that of the tangent; the magnitude does not change. A factor of 1, the force tangent to the slip surface, is a common assumption for extensible reinforcement in embankments on soft ground.

5.4 Stabilizing piles

A row of piles resists with its shear force V_u divided by the spacing, applied where the pile crosses the slip surface, horizontally (perpendicular to a vertical pile) or parallel to the slip surface. The shear resistance may be constant along the pile or decrease linearly to zero at the pile tip below a given proportion of the length. The resistance is not divided by the factor of safety. The shear resistance can be taken from a single-pile analysis: the shear demand of the pile for a soil movement above a slip surface at the depth where the slip surface crosses the pile (Reese, Wang and Fouse 1992).

The shear resistance may also be given as a **shear demand table** from the single-pile analysis: the shear of one pile for slip surfaces at several depths below the pile head. Every trial slip surface then uses the shear at its own depth z, interpolated linearly in the table, multiplied by a factor k and divided by the spacing, $T = k\ V(z)\ /\ b$. Above the first row of the table the shear decreases linearly to zero at the pile head; below the last row it keeps the last value. A factor below 1 accounts, for example, for the moment arm of a shear demand that acts above the slip surface. The results list, for each pile, the depth of the slip surface and the active and passive forces on the pile that correspond to the design factor of safety.

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6 Verification

6.1 Methodologies

The factor of safety is compared with a requirement according to the verification methodology of the project (Table 6.1). The design situation (permanent, transient, accidental, seismic) selects the requirement or the partial factors.

Table 6.1 — Verification methodologies

Methodology	Check
Factor of safety	$F \geq F_{req}$; the default requirements are 1.5 (permanent), 1.3 (transient), 1.1 (accidental and seismic).
Limit states	Strength divided by the factors on tan φ and c; utilization $F_{req}/F \leq 100$ %.
EN 1997	Partial factors of the selected design approach on the strength and on the actions (Table 6.2) in permanent and transient situations, 1.0 in accidental and seismic situations; utilization $\gamma_R/F \leq 100$ %.
LRFD 2012	Weight of the soil and permanent surcharges factored as earth surcharge (ES, minimum where favourable, maximum where unfavourable), variable surcharges as live load (LL), water unfactored; resistance factor φ_{SS} on the stability (AASHTO 2012). Service I: ES 1.0, LL 1.0, $\varphi_{SS} = 0.65$; Extreme I: ES 0.75–1.5, LL up to 1.75, $\varphi_{SS} = 0.9$. Utilization $1/(\varphi_{SS} F) \leq 100$ %.
Chinese standards	$F \geq F_{req}$ by situation and slip surface (GB 50330-2013): 1.35 for polygonal and 1.30 for circular surfaces in permanent and transient situations, 1.15 in accidental and seismic situations.

Table 6.2 — Partial factors of EN 1997 used for slopes

Design approach	tan φ'	c'	c_u	permanent action (unfav./fav.)	variable action	γ_R
DA1, combination 2	1.25	1.25	1.4	1.0 / 1.0	1.3	1.0
DA2	1.0	1.0	1.0	1.35 / 1.0	1.5	1.1
DA3	1.25	1.25	1.4	1.0 / 1.0	1.3	1.0

The weight of the soil in a slice is a permanent action: it is unfavourable when $\tan\varphi \cos\alpha - \sin\alpha < 0$ (α = inclination of the slice base, φ = friction angle at the base) and favourable otherwise. The default values can be changed in the program options for each methodology.

6.2 Sums of forces and moments

The results summary gives, besides the factor of safety, the driving and resisting quantities of the critical surface. For a circle they are taken about its centre: the sliding moment M_a is the moment of the weights, surcharges, inertia forces and water forces about the centre, the resisting moment is $M_p = F\,M_a$, and the sums of the active and passive forces along the surface are $F_a = M_a / R$ and $F_p = F\,F_a$. The sliding moment does not depend on the method, so on one circle the methods differ only in M_p and F_p. A polygonal surface has no centre: F_a is the sum of the mobilised shear forces T on the slice bases and $F_p = F\,F_a$ the available shear resistance.

6.3 Surplus sliding force

For a slip surface whose factor of safety is below the design value F_d, the surplus sliding force is the horizontal force at the toe (or at the crest) that would raise the factor of safety to F_d. It is the interslice force that remains at the end of the mass when the force equilibrium is solved with the strength divided by F_d, and indicates the force that a retaining structure or reinforcement has to provide.

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7 Verification of the implementation

7.1 Approach

The implementation is checked against closed-form solutions and against independent recalculation of the program output. The checks below can be repeated by the user with the inputs given; the numbers are computed by Solstability 1.0.1 from those inputs. Differences of a few tenths of a percent arise from the division of the sliding mass into slices, which closed-form solutions do not have.

7.2 Infinite slope

For a dry cohesionless soil and a slip surface parallel to the ground surface, the factor of safety of an infinite slope is $F = \tan\varphi \,/\, \tan\beta$. A slope of inclination $\beta = 20°$ and 200 m length in sand with $\varphi = 30°$ and $c = 0$ is analysed with a polygonal slip surface 2 m below and parallel to the surface, with its ends on the toe and the crest. The short end segments make the surface slightly different from an infinite one.

Table 7.1 — Infinite slope, factor of safety

Solution	F
tan φ / tan β	1.5863
Spencer	1.5922
Janbu	1.5918
Morgenstern-Price	1.5893

7.3 Undrained clay

For a soil with $\varphi = 0$ the factor of safety of a circle is the ratio of the resisting moment of the cohesion to the driving moment of the weight about the centre, $F = c\,L\,R \,/\, (W\,e)$, where L is the length of the arc, W the weight of the sliding mass and e the horizontal distance of its centroid from the centre. A slope 6 m high at 45° in clay with $c_u = 30$ kPa and $\gamma = 18$ kN/m^3 is analysed with the circle of centre (22, 12) m and radius 13 m. W, e and L are computed from the exact geometry of the circle; Bishop and Fellenius coincide for $\varphi = 0$ and differ from the exact value only by the slicing.

Table 7.2 — Undrained clay, factor of safety

Solution	F
Exact moment ratio	1.7223
Bishop	1.7275
Fellenius	1.7275

7.4 Recalculation from the slice table

The factors of safety of Fellenius and Bishop can be recalculated by hand from the slice table of the results (b, α, l, c, φ, W, U) with (3.2) and (3.3). For the embankment of Example 1 of the User Manual with its input circle (20 slices) the recalculation gives the values below. The small differences come from slices whose base crosses two soils: the program integrates c and φ along the base, the table lists their mean values.

Table 7.3 — Recalculation from the slice table

Method	Program	Recalculated
Fellenius	1.6247	1.6238
Bishop	1.7259	1.7249

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7.5 Agreement between methods

On the critical circle of Example 1 the methods that satisfy complete equilibrium give practically the same factor of safety, Bishop is within a fraction of a percent of them, and Fellenius is lower, as expected for a frictional soil (Table 7.4).

Table 7.4 — Methods on the critical circle of Example 1

Method	F
Bishop	1.543
Spencer	1.539
Janbu	1.539
Morgenstern-Price	1.539
Fellenius	1.472

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