

LATERAL PILE ANALYSIS · P-Y METHOD

Solpile

Technical Reference

Formulation, constitutive models, numerical solution
and verification.

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1 Scope and notation

1.1 Purpose

This reference describes the engineering model and the numerical methods implemented in Solpile 1.0.0: the idealisation of the pile and the soil, the governing equation and its finite-difference solution, the p-y models of soil and rock, the moment-curvature analysis of pile sections, the special analyses, and the verification of the implementation. It is intended to let an engineer judge whether the program is appropriate for a problem and to check its results independently. The use of the program is described in the User Manual (SPL-SP-002).

The p-y models are empirical relations published in the technical literature. For each model this reference gives the equations as they are evaluated by Solpile and the publication on which the model is based. The publications describe the test programmes, the range of validity and the limitations of each model; users should consult them when a model is applied near or beyond the conditions for which it was developed.

1.2 Notation

Symbols are defined where they first appear. The principal symbols are listed below. Equations are written in consistent units; where a correlation is dimensional, the units are stated with the equation.

Table 1.1 — Principal symbols

Symbol	Meaning	Symbol	Meaning
b	pile width used for the p-y curve	p	soil reaction per unit length
c_u, c	undrained shear strength, cohesion	p_u	ultimate soil reaction
E_s	secant p-y modulus, p/y	P	axial thrust (compression +)
EI, R	bending stiffness	S	slope, dy/dx
h	increment length	V, M	shear force, bending moment
k	initial p-y modulus gradient	W	distributed lateral load
K_0, K_a, K_p	earth-pressure coefficients	x	depth below the pile head
L	pile length	y	lateral deflection
N	number of increments; number of load cycles	y_{50}	deflection at $p = 0.5p_u$
γ'	effective unit weight	ϵ_{50}	strain at 50 % of peak deviator stress
σ_v	vertical effective stress	ϕ'	effective friction angle

2 Structural idealisation

2.1 Assumptions

The pile is treated as a beam-column of length L loaded in one plane (Figure 2.1). The following assumptions apply:

- (a) plane sections remain plane; deflections and rotations are small; shear deformation is neglected except where Section 4.6 applies;
- (b) the bending stiffness may vary along the pile and, for nonlinear sections, with the bending moment;
- (c) the axial thrust P is constant along the pile;
- (d) the soil acts on the pile as a continuous bed of independent nonlinear springs, each described by a p - y curve (Winkler idealisation);
- (e) loads are static; cyclic loading is represented through cyclic p - y curves.

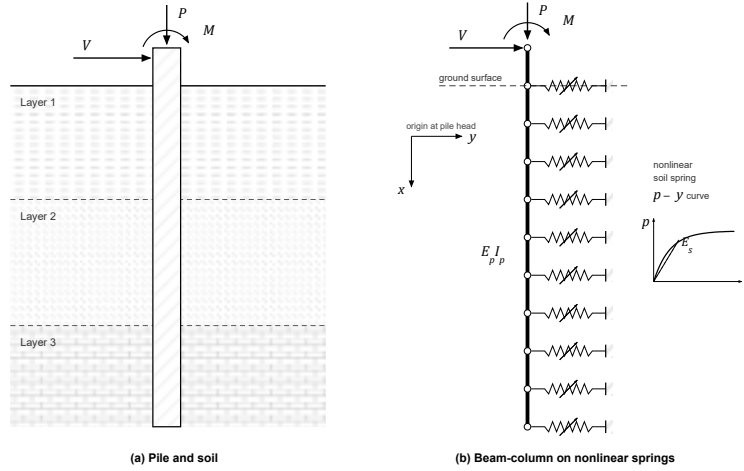


Figure 2.1 — Idealisation of the pile-soil system

2.2 Governing equation

Equilibrium of a beam-column element with axial thrust P , distributed lateral load W and soil reaction p gives

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 y}{dx^2} \right) + P \frac{d^2 y}{dx^2} - p - W = 0 \quad (2.1)$$

where p is the soil reaction, which acts against the deflection. Writing the reaction with the secant modulus of the p - y curve,

$$p = -E_s y, \quad E_s = E_s(y, x) \quad (2.2)$$

the governing equation becomes

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 y}{dx^2} \right) + P \frac{d^2 y}{dx^2} + E_s y = W \quad (2.3)$$

The secant modulus depends on the deflection, so the equation is nonlinear and is solved iteratively (Section 3.3). The internal forces follow from the deflected shape:

$$S = \frac{dy}{dx}, \quad M = EI \frac{d^2 y}{dx^2}, \quad V = \frac{dM}{dx} + P \frac{dy}{dx} \quad (2.4)$$

2.3 Boundary conditions

Two conditions are prescribed at the pile head ($x = 0$), according to the type of head condition:

Table 2.1 — Head boundary conditions

Type	Condition 1	Condition 2
1	$V(0) = V_t$	$M(0) = M_t$
2	$V(0) = V_t$	$S(0) = S_t$
3	$V(0) = V_t$	$M(0) = k_\theta \cdot S(0)$
4	$y(0) = y_t$	$M(0) = M_t$
5	$y(0) = y_t$	$S(0) = S_t$

At the tip ($x = L$) the moment is zero and the shear is zero, unless tip resistance is modelled (Section 4.4), in which case the tip shear and moment are those of the tip springs.

3 Numerical solution

3.1 Finite-difference discretisation

The pile is divided into N increments of equal length $h = L/N$ with nodes $m = 0$ (head) to N (tip). Two imaginary nodes are added beyond each end to express the boundary conditions (Figure 3.1). The unknowns are the deflections of the $N + 5$ nodes.

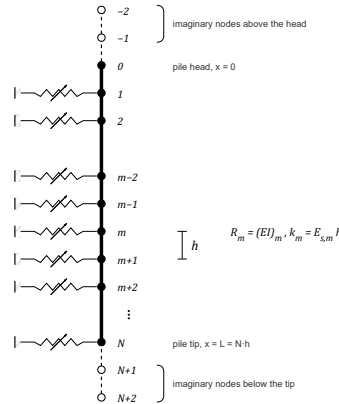


Figure 3.1 — Nodes of the finite-difference model

With $R_m = (EI)_m$, central differences applied to (2.3) at node m give

$$R_{m-1} y_{m-2} + (-2R_{m-1} - 2R_m + Ph^2) y_{m-1} + (R_{m-1} + 4R_m + R_{m+1} - 2Ph^2 + E_{s,m} h^4) y_m + (-2R_m - 2R_{m+1} + Ph^2) y_{m+1} + R_{m+1} y_{m+2} = W_m h^4 \quad (3.1)$$

At the end nodes the stiffness of the imaginary nodes is taken equal to that of the adjacent real node.

3.2 Boundary equations

The head conditions are written with central differences at node 0:

$$M_0 = \frac{R_0}{h^2} (y_{-1} - 2y_0 + y_1), \quad S_0 = \frac{y_1 - y_{-1}}{2h} \quad (3.2)$$

$$V_0 = \frac{1}{2h^3} [-R_0 y_{-2} + 2R_0 y_{-1} + (R_1 - R_0) y_0 - 2R_1 y_1 + R_1 y_2] + \frac{P}{2h} (y_1 - y_{-1}) \quad (3.3)$$

For type 3 the moment condition becomes $M_0 - k_\theta S_0 = 0$; for types 4 and 5 the first condition is $y_0 = y_t$. The same expressions written at node N give the tip conditions. The system of $N + 5$ equations is banded and is solved directly.

3.3 Iteration

The nonlinearity of the soil (and, for nonlinear sections, of the bending stiffness) is treated by secant iteration:

1. Initial secant moduli are taken from the initial branch of each p-y curve.
2. The linear system is solved for the deflections.
3. At every node the secant modulus is updated from the p-y curve at the new deflection, $E_{s,m} = p(y_m)/y_m$. For nonlinear sections the bending stiffness is updated from the moment-curvature relation at the new moment (Section 6.7).
4. Steps 2 and 3 are repeated until the largest change of nodal deflection between two iterations is smaller than the convergence tolerance and, for nonlinear sections, the bending stiffness has settled.

The iteration of a load case stops without a result when the iteration limit is reached, when the head deflection exceeds the deflection limit, or when the system matrix becomes singular because the soil is fully mobilised along the pile. These conditions indicate that the load exceeds the lateral capacity of the pile-soil system or that the model is not stable under the axial thrust.

Curves whose secant modulus is unbounded at zero deflection (for example the cubic-parabola clay curves) are given a finite modulus in the range of very small deflections, so that the first iteration and nodes with negligible deflection are well conditioned. This does not affect the curves in the range of practical deflections.

3.4 Recovery of results

After convergence, slope, moment and shear at every node are obtained by central differences:

$$S_m = \frac{y_{m+1} - y_{m-1}}{2h}, \quad M_m = R_m \frac{y_{m-1} - 2y_m + y_{m+1}}{h^2}, \quad V_m = \frac{M_{m+1} - M_{m-1}}{2h} + P S_m \quad (3.4)$$

The soil reaction is $p_m = -E_{s,m} y_m$; the curvature is M_m/R_m . The truncation error of the scheme is of second order in h (Section 8.3).

4 Loads and structural effects

4.1 Axial thrust

The axial thrust enters (2.3) through the term $P \cdot d^2y/dx^2$ (P-Δ effect) when that option is active. It also governs the moment-curvature relation of nonlinear sections (Chapter 6). Solpile rejects a thrust that lies outside the nominal axial capacity of a section.

4.2 Distributed lateral load

A distributed load $W(x)$ is given by points (depth, intensity) with linear variation between them. Its nodal value W_m enters the right-hand side of (3.1).

4.3 Concentrated loads

A concentrated lateral force at a depth between two nodes is distributed to these nodes in proportion to its distance from each (linear shape functions). A concentrated moment is represented by a pair of opposite forces at a distance h , centred at the point of application.

4.4 Pile-tip resistance

When tip resistance is modelled, the shear condition at the tip includes a spring force $V_{\text{tip}} = K_V(y_N) \cdot y_N$ and the moment condition a spring moment $M_{\text{tip}} = K_M(\theta_N) \cdot \theta_N$, where the secant stiffnesses K_V and K_M are taken from the user tables of shear against tip deflection and moment against tip rotation. The spring forces are updated in each iteration.

4.5 Battered pile

For a pile inclined at a batter angle ψ , the soil properties at a node are taken at the vertical depth $x \cos \psi$. When the pile head is above the ground, its height h_s is a vertical distance, so a node at distance x from the head along the pile lies at the vertical depth $x \cos \psi - h_s$ below the ground. The effect of batter on the ultimate soil resistance is described in Section 5.19.

4.6 Shear deformation

For sections of the type *Elastic Section* the shear deformation can be included. The pile is then analysed as a Timoshenko beam in which the slope of the axis differs from the rotation of the cross-section by the shear strain $V/(\kappa GA)$, with $G = 0.4E$ and a shear correction factor κ that depends on the shape: 5/6 for rectangular and for a general shape entered as area and inertia, 0.9 for solid circular and embedded-pole sections, and 0.5 for pipes. For the H-pile, AISI, C, T and box shapes the correction uses the web area of the shape, $\kappa = d t_w/A$ about the strong axis and the flange area about the weak axis, and it is applied only when those plate dimensions are entered. Elastic sections with a specified moment capacity and nonlinear sections are analysed without shear deformation. The additional flexibility is significant only for short, stocky piles.

5 Soil response: p-y curves

5.1 General

A p-y curve gives the soil reaction p (force per unit length of pile) as a function of the lateral deflection y at one depth. Solpile generates a curve at every node from the soil material of the layer that contains the node, the pile width b at that node, the depth and the vertical effective stress. Representative curves of the main models are shown in Figure 5.1.

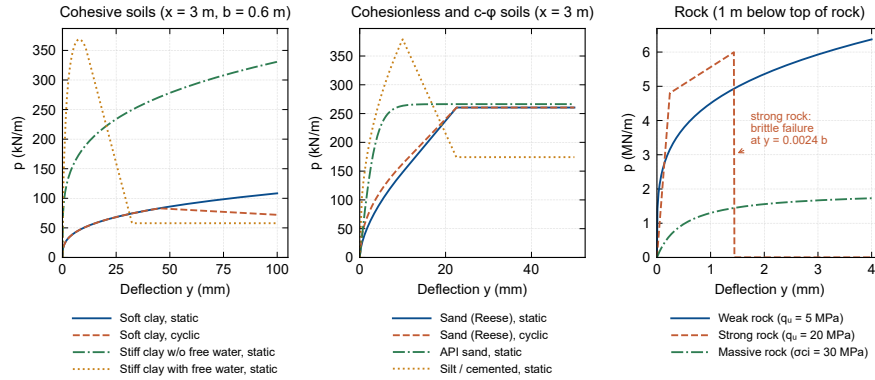


Figure 5.1 — p-y curves of the main models, computed by Solpile for $b = 0.6$ m

5.1.1 Effective stress

The vertical effective stress is integrated from the ground surface:

$$\sigma'_v(x) = \int_0^x \gamma'(\zeta) d\zeta, \quad \gamma' = \gamma \text{ above the water table, } \gamma' = \gamma_{sat} - \gamma_w \text{ below} \quad (5.1)$$

with $\gamma_w = 9.81 \text{ kN/m}^3$. In the expressions for the ultimate resistance of the shallow soil wedge, γ' denotes the average effective unit weight σ'_v/x . Depths in this chapter are measured below the ground surface.

5.1.2 Static and cyclic curves

Each model provides a static curve and, where the model defines one, a cyclic curve. The choice applies to the whole analysis. Cyclic curves represent the degradation of resistance under repeated loading; they are not the result of a cyclic analysis.

5.1.3 Asymmetric curves

Without ground slope the curves are symmetric: $p(-y) = -p(y)$. With ground slope or pile batter the ultimate resistance of the shallow wedge differs for loading towards and away from the slope, and the curve is evaluated with the resistance that applies to the direction of the deflection (Section 5.19).

5.2 Soft clay

Basis: Matlock (1970); the empirical factor J is 0.5 for the model *Soft Clay (Matlock)* and is entered for *API Soft Clay* (API, 2000). The ultimate resistance is the smaller of the wedge and flow-around values:

$$p_u = \min \left[\left(3 + \frac{\gamma' x}{c_u} + \frac{J x}{b} \right) c_u b, 9 c_u b \right] \quad (5.2)$$

$$y_{50} = 2.5 \varepsilon_{50} b \quad (5.3)$$

Static loading:

$$p = 0.5 p_u \left(\frac{y}{y_{50}} \right)^{1/3} \quad (y \leq 8 y_{50}), \quad p = p_u \quad (y > 8 y_{50}) \quad (5.4)$$

Cyclic loading: the curve follows the static curve up to $0.72 p_u$ at $y = 3 y_{50}$ (precisely $0.5 \cdot 3^{1/3} = 0.7211$). Below the transition depth

$$x_r = \frac{6 c_u b}{\gamma' b + J c_u} \geq 2.5 b \quad (5.5)$$

the resistance remains $0.72 p_u$ beyond $3 y_{50}$. Above it, the resistance decreases linearly from $0.72 p_u$ at $3 y_{50}$ to $0.72 p_u \cdot x_r / x_t$ at $15 y_{50}$ and remains constant thereafter.

5.3 Stiff clay with free water

Basis: Reese, Cox and Koop (1975). The ultimate resistance is

$$p_c = \min \left[2 c_a b + \gamma' b x + 2.83 c_a x, 11 c b \right] \quad (5.6)$$

where c is the undrained shear strength at the depth considered and c_a the average value over the depth of the wedge, which equals c in a layer of uniform strength. With $y_{50} = \varepsilon_{50} b$ and the empirical coefficients $A_s(x/b)$ (static) and $A_c(x/b)$ (cyclic), which Solpile evaluates from tabulated representations of the published charts, the static backbone curve is

$$p = 0.5 p_c \left(\frac{y}{y_{50}} \right)^{0.5} \quad (y \leq A_s y_{50}) \quad (5.7)$$

$$p = 0.5 p_c \left(\frac{y}{y_{50}} \right)^{0.5} - 0.055 p_c \left(\frac{y - A_s y_{50}}{A_s y_{50}} \right)^{1.25} \quad (A_s y_{50} < y \leq 6 A_s y_{50}) \quad (5.8)$$

$$p = 0.5 p_c \sqrt{6A_s} - 0.411 p_c - \frac{0.0625}{y_{50}} p_c (y - 6A_s y_{50}) \quad (6A_s y_{50} < y \leq 18A_s y_{50}) \quad (5.9)$$

$$p = p_c \left(0.5 \sqrt{6A_s} - 0.75 A_s - 0.411 \right) \quad (y > 18A_s y_{50}) \quad (5.10)$$

For cyclic loading, with $y_p = 4.1A_s y_{50}$,

$$p = A_c p_c \left[1 - \left| \frac{y - 0.45 y_p}{0.45 y_p} \right|^{2.5} \right] \quad (y \leq 0.6 y_p) \quad (5.11)$$

$$p = 0.936 A_c p_c - \frac{0.085}{y_{50}} p_c (y - 0.6 y_p) \quad (0.6 y_p < y \leq 1.8 y_p) \quad (5.12)$$

$$p = p_c (0.936 A_c - 0.102 \cdot 4.1 A_s) \quad (y > 1.8 y_p) \quad (5.13)$$

In both cases the curve is limited by the initial straight line $p = kxy$, where k is the initial modulus gradient: $p = \min(kxy, \text{backbone})$.

5.4 Stiff clay without free water

Basis: Welch and Reese (1972). A modified form adds an initial straight branch defined by the modulus gradient k . With

$$p_u = \min \left[\left(3 + \frac{\gamma' x}{c} + 0.5 \frac{x}{b} \right) c b, 9 c b \right], \quad y_{50} = 2.5 \varepsilon_{50} b \quad (5.14)$$

the static curve is

$$p = 0.5 p_u \left(\frac{y}{y_{50}} \right)^{1/4} \quad (y \leq 16 y_{50}), \quad p = p_u \quad (y > 16 y_{50}) \quad (5.15)$$

For cyclic loading with N cycles the deflection at a given resistance increases, and the curve becomes

$$p = p_u \left(\frac{y}{y_{50} (16 + 9.6 \log_{10} N)} \right)^{1/4} \leq p_u \quad (5.16)$$

which reduces to (5.15) for $N = 1$. In the **modified** model the curve is limited by the initial line $p = kxy$.

5.5 ISO clay

Basis: ISO 19901-4 (2025), A.8.5.2, which adopts the framework of Jeanjean et al. (2017) for monotonic loading and of Zhang et al. (2017) for cyclic loading. The undrained shear strength s_u is the direct simple shear (DSS) strength; a strength measured by another type of test is converted to an equivalent DSS strength by a fixed factor for the selected test type. The ultimate resistance is

$$p_u = N_p s_u D \quad (5.17)$$

The bearing capacity factor for flow around the pile at depth is $N_{pd} = 9 + 3\alpha_{ave}$, where α_{ave} is the pile-soil adhesion factor $\alpha = 0.5\psi^{-0.5}$ ($\psi \leq 1$) or $0.5\psi^{-0.25}$ ($\psi > 1$), $\psi = s_u/\sigma'_v$, $\alpha \leq 1$ (API, 2000), averaged over three points of the layer: its top, the smallest of its bottom, $20D$ and the pile tip, and the point midway between them. Near the surface the wedge factor is

$$N_{p0} = \min \left\{ \left(11 + \alpha_{ave} \right) - 8.78 \left[1 - \left(\frac{z}{d} \right)^{0.6} \right]^{1.35}, N_{pd} \right\}, \quad d = \max \left(14.5, 16.8 - 2.3 \log_{10} \frac{s_{u0}}{s_{u1} D} \right) \quad (5.18)$$

where s_{u0} and s_{u1} are the strength at the top of the layer and its gradient with depth ($d = 14.5$ in a layer of uniform strength), and the bracket is zero below $z = dD$. Without a gap behind the pile, $N_p = \min(2N_{p0}, N_{pd})$. With a gap,

$$N_p = \min \left(\frac{\sigma'_v}{s_u} + C_w N_{p0}, N_{pd} \right), \quad C_w = 1 + \left(\frac{s_{u,TE}}{s_{u,DSS}} - 1 \right) \frac{N_{pd} - N_p^*}{N_{pd} - N_{p0}(z=0)} \quad (5.19)$$

where $N_p^* = \sigma'_v/s_u + N_{p0}$ is the factor before the correction for strength anisotropy, which is applied only when $N_p^* < N_{pd}$, and $s_{u,TE}/s_{u,DSS}$ is the ratio of triaxial extension to DSS strength entered with the material (default 0.9). With the gapping option *Estimated by program*, a gap is assumed when every layer from the ground surface to a depth of 10 m, or to the pile tip if shallower; is ISO clay with $s_u \geq 15$ kPa at its top and bottom; otherwise no gap is assumed. When the first layer is sand thicker than D , $N_p = N_{pd}$.

The monotonic curve follows the default normalised p-y curves of ISO 19901-4, which give y/D at $p/p_u = 0, 0.05, 0.2, 0.3 \dots 0.9, 0.975$ and 1 for clays with a plasticity index above or below 30 % and an over-consolidation ratio of 2, 4 and 10. Solpile interpolates linearly in OCR (values outside 2 to 10 take the nearest column), joins the points by straight lines and keeps $p = p_u$ beyond the last point. For cyclic loading each point of the monotonic curve is modified (Zhang et al., 2017):

$$p_{cy} = (a_p - b_p \ln N_{eq}) p_{mo}, \quad y_{cy} = (1.2 - b_y \ln N_{eq}) y_{mo}, \quad N_{eq} = \left(\frac{2}{1 - h_f} \right)^g, \quad 1 \leq N_{eq} \leq 25 \quad (5.20)$$

with $h_f = p_{mo}/p_u - (z/z_{rot})^2$ above the depth of rotation z_{rot} (default 15D) and $h_f = p_{mo}/p_u - 1$ below it, limited to $-1 \dots 1$. The constants g , a_p , b_p and b_y are those of ISO 19901-4 for the selected design condition: Gulf of Mexico clay, North Sea soft clay or North Sea stiff clay.

5.6 Sand (Reese)

Basis: Reese, Cox and Koop (1974). With $\alpha = \varphi'/2$, $\beta = 45^\circ + \varphi'/2$, $K_0 = 0.4$ and $K_a = \tan^2(45^\circ - \varphi'/2)$, the resistances of the shallow wedge and of flow around the pile at depth are

$$p_{st} = \gamma' x \left[\frac{K_0 x \tan \varphi' \sin \beta}{\tan(\beta - \varphi') \cos \alpha} + \frac{\tan \beta}{\tan(\beta - \varphi')} (b + x \tan \beta \tan \alpha) + K_0 x \tan \beta (\tan \varphi' \sin \beta - \tan \alpha) - K_a b \right] \quad (5.21)$$

$$p_{sd} = K_a b \gamma' x (\tan^8 \beta - 1) + K_0 b \gamma' x \tan \varphi' \tan^4 \beta \quad (5.22)$$

and $p_s = \min(p_{st}, p_{sd})$. The curve consists of four parts (Figure 5.1):

1. an initial straight line $p = kxy$;
2. a parabola $p = Cy^{1/n}$ between the intersection with the initial line, at $y_k = (C/kx)^{n/(n-1)}$, and point m ;
3. a straight line from point m ($y_m = b/60$, $p_m = B p_s$) to point u ($y_u = 3b/80$, $p_u = A p_s$);
4. a horizontal line $p = p_u$ beyond y_u .

$$m = \frac{p_u - p_m}{y_u - y_m}, \quad n = \frac{p_m}{m y_m}, \quad C = \frac{p_m}{y_m^{1/n}} \quad (5.23)$$

When the initial line meets the curve beyond y_m , the parabola is omitted. The coefficients A and B depend on x/b and on the loading (Table 5.1); Solpile interpolates linearly between the tabulated values and uses the last value beyond $x/b = 5$.

Table 5.1 — Coefficients A and B for sand (Reese et al., 1974)

x/b	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	≥ 5.0
A_s (static)	2.83	2.48	2.11	1.76	1.48	1.23	1.06	0.90	0.88
B_s (static)	2.14	1.83	1.54	1.27	1.05	0.86	0.71	0.53	0.50
A_c (cyclic)	0.75	0.93	1.07	1.11	1.03	0.98	0.94	0.90	0.88
B_c (cyclic)	0.53	0.72	0.83	0.86	0.82	0.77	0.70	0.55	0.55

5.7 API sand

Basis: O'Neill and Murchison (1983); API (2000). The ultimate resistance is

$$p_u = \min \left[(C_1 x + C_2 b) \gamma' x, C_3 b \gamma' x \right] \quad (5.24)$$

$$C_1 = \frac{\tan^2 \beta \tan \alpha}{\tan(\beta - \varphi')} + K_0 \left[\frac{\tan \varphi' \sin \beta}{\cos \alpha \tan(\beta - \varphi')} + \tan \beta (\tan \varphi' \sin \beta - \tan \alpha) \right], \quad C_2 = K_p - K_a, \quad C_3 = K_a (\tan^8 \beta - 1) + K_0 \tan \varphi' \tan^4 \beta \quad (5.25)$$

with the symbols of Section 5.6 and $K_p = \tan^2(45^\circ + \varphi'/2)$. The curve is

$$p = A p_u \tanh \left(\frac{k x}{A p_u} y \right), \quad A = \max \left(3 - 0.8 \frac{x}{b}, 0.9 \right) \text{ (static)}, \quad A = 0.9 \text{ (cyclic)} \quad (5.26)$$

5.8 Enhanced sand option

Basis: Kallehave, LeBlanc Thilsted and Liingaard (2012). The curves of Section 5.6 and Section 5.7 start with an initial modulus that grows linearly with depth, $E_{py} = kx$, derived from tests on piles of about 0.6 m diameter. With the enhanced option the initial modulus is

$$E_{py} = k z_{ref} \left(\frac{x}{z_{ref}} \right)^m \left(\frac{b}{D_{ref}} \right)^n, \quad z_{ref} = 2.5 \text{ m (98 in)}, \quad D_{ref} = 0.61 \text{ m (24 in)} \quad (5.27)$$

where x is the actual depth. Kallehave et al. (2012) give $m = 0.6$ and $n = 0.5$ for the API sand curve; these are the defaults for API sand, and $m = 0.6$, $n = 0.33$ for sand (Reese). In the API curve E_{py} replaces kx in (5.26). In the Reese curve it replaces kx in the initial line, and the deflections y_m and y_u are multiplied by kx/E_{py} . The option also accepts K_0 (default 0.4) and the wedge angle α (default $\varphi'/2$, less than 45°), which then sets $\beta = 45^\circ + \alpha$, $K_a = \tan^2(45^\circ - \alpha)$ and, in API sand, $K_p = \tan^2 \beta$. An entry of zero selects the default.

5.9 Small-strain sand

Basis: Hanssen (2015), with the modulus degradation of Hardin and Drnevich (1972) and the correlations of Wichtmann and Triantafyllidis (2009, 2013). The curve starts with the small-strain stiffness $4G_{max}$ and approaches the API ultimate resistance $A p_u$ of (5.24) and (5.26). With the reference deflection and the normalised deflection

$$y_r = \frac{A p_u}{4 G_{max}}, \quad c = \frac{y}{y_r} \quad (5.28)$$

the lower branch is

$$p = \frac{4 G_{max} y}{1 + c (1 + a e^{-bc})} \quad (c \leq 1) \quad (5.29)$$

where a and b are shape constants. Beyond y_r , Solpile continues the curve with a smooth branch that matches the value and the slope of the lower branch at $c = 1$ and tends asymptotically to $A p_u$. G_{max} , a and b are obtained in one of two ways:

- Option 1: G_{max} is the small-strain shear modulus G_0 entered with the material, and a and b are entered (defaults 0.8 and 1.0).
- Option 2: when G_0 is zero and the void ratio e and the uniformity coefficient C_u are entered, G_{max} follows from Wichtmann and Triantafyllidis (2009) and the shape constants from Wichtmann and Triantafyllidis (2013):

$$G_{max} = A \frac{(a_1 - e)^2}{1 + e} p_a^{1-n} p'^n, \quad A = 1563 + 3.13 C_u^{2.98}, \quad a_1 = 1.94 e^{-0.066 C_u}, \quad n = 0.40 C_u^{0.18}, \quad a = 1.07 \ln C_u, \quad b = 1 \quad (5.30)$$

with $p_a = 100 \text{ kPa}$ (14.7 psi) and the mean effective stress $p' = \sigma'_v(1 + 2K_0)/3$, $K_0 = 0.4$. When neither G_0 nor e and C_u are entered, the layer is computed with the API sand curve of (5.26) and the model check issues the warning SSS-G0-ZERO.

5.10 Liquefied sand

5.10.1 Liquefied sand (Rollins)

Basis: Rollins et al. (2005), from full-scale tests in liquefied sand. The curve for a 0.3 m pile is expressed with y in millimetres, z in metres and p in kN/m:

$$p_{0.3} = A (B y)^C \leq 15 \text{ kN/m}, \quad A = 3 \times 10^{-7} (z + 1)^{6.05}, \quad B = 2.80 (z + 1)^{0.11}, \quad C = 2.85 (z + 1)^{-0.41} \quad (5.31)$$

with y limited to 150 mm. For other diameters the reaction is scaled by

$$p = P_d p_{0.3}, \quad P_d = 3.81 \ln b + 5.6 \quad (0.3 \leq b \leq 2.6 \text{ m}), \quad P_d = b/0.3 \quad (b < 0.3 \text{ m}) \quad (5.32)$$

with P_d not greater than 9.24. For a liquefied layer below other layers the depth z is replaced by a normalised depth based on the vertical effective stress. The curves are recommended to a depth of 6 m.

5.10.2 Liquefied sand, hybrid model

Basis: Franke and Rollins (2013). The curve is the lower of the dilative curve above and a residual-strength curve computed with the soft-clay model (Section 5.2, static, $J = 0.5$) using the residual shear strength S_{ur} and ε_{50} :

$$p = \min \left[p_{dilative}(y), p_{residual}(y) \right] \quad (5.33)$$

When S_{ur} is not entered it is estimated from the corrected blow count $(N_1)_{60-cs}$ with a residual-strength correlation; it is zero for blow counts below 5. Enter S_{ur} and ε_{50} explicitly where site-specific values are available.

5.11 Cemented soils with cohesion and friction

Basis: the $c-\phi$ procedure of Reese and Van Impe (2001). The frictional and cohesive components of the ultimate resistance are computed separately:

$$p_{u\phi} = \min \left[\sigma'_v (C_1 x + C_2 b), \sigma'_v C_3 b \right] \quad (5.34)$$

$$p_{uc} = \min \left[\left(3 + \frac{\gamma' x}{c} + 0.5 \frac{x}{b} \right) c b, 9 c b \right] \quad (5.35)$$

with C_1, C_2, C_3 as in (5.25). The curve is constructed like the sand curve of Section 5.6: an initial line $p = kxy$, a parabola, a straight line from point m ($y_m = b/60$) to point u ($y_u = 3b/80$) and a horizontal line. The resistance at point m combines the frictional and the cohesive components; at point u only the frictional component, $A p_{u\phi}$, remains. The cohesive component therefore contributes to the peak but is not retained at large deflection. When the initial line meets the straight segment beyond y_m , the parabola is omitted. Both c and ϕ' must be greater than zero.

5.12 Rock

5.12.1 Weak rock

Basis: Reese (1997). With x_r the depth below the top of the rock and the strength reduction factor $\alpha_r = 1 - (2/3)(RQD/100)$, bounded to $1/3-1$:

$$p_{ur} = \alpha_r q_u b \left(1 + 1.4 \frac{x_r}{b} \right) \quad (x_r \leq 3b), \quad p_{ur} = 5.2 \alpha_r q_u b \quad (x_r > 3b) \quad (5.36)$$

$$M_{ir} = k_{ir} E_{ir}, \quad k_{ir} = 100 + 400 \frac{x_r}{3b} \quad (x_r \leq 3b), \quad k_{ir} = 500 \quad (x_r > 3b) \quad (5.37)$$

With $y_{rm} = k_{rm} b$, the curve is

$$p = M_{ir} y \quad (y \leq y_A), \quad p = \frac{p_{ur}}{2} \left(\frac{y}{y_{rm}} \right)^{1/4} \quad (y_A < y \leq 16 y_{rm}), \quad p = p_{ur} \quad (y > 16 y_{rm}) \quad (5.38)$$

$$y_A = \left(\frac{p_{ur}}{2 y_{rm}^{1/4} M_{ir}} \right)^{4/3} \quad (5.39)$$

5.12.2 Strong rock

Basis: Reese and Nyman (1978) for vuggy limestone. With $s_u = q_u/2$ and $p_u = b s_u$, the curve is bilinear: slope $2000s_u$ up to $y = 0.0004b$, then slope $100s_u$ up to p_u . Beyond the deflection at which p_u is reached ($y = 0.0024b$) the rock is assumed to fail in a brittle manner, and the resistance drops to a small residual value.

5.12.3 Massive rock

Basis: Liang, Yang and Nusairat (2009), with the rock mass described by the generalised Hoek–Brown criterion (Hoek et al., 2002). The curve is hyperbolic:

$$p = \frac{y}{\frac{1}{K_i} + \frac{y}{p_u}} \quad (5.40)$$

The initial modulus is

$$K_i = E_m \frac{D}{D_{ref}} e^{-2\nu} \left(\frac{E_p I_p}{E_m D^4} \right)^{0.284}, \quad D_{ref} = 0.305 \text{ m} \quad (5.41)$$

where E_m is the rock mass modulus, entered directly or estimated from the intact modulus and the geological strength index GSI. The Hoek–Brown parameters are

$$m_b = m_i e^{(GSI - 100)/28}, \quad s = e^{(GSI - 100)/9}, \quad a = 0.5 \quad (GSI \geq 25), \quad a = 0.65 - GSI/200 \quad (GSI < 25) \quad (5.42)$$

At the confining stress of interest the criterion is replaced by the instantaneous Mohr–Coulomb parameters ϕ' and c' (Hoek, 1990). The ultimate resistance is the smaller of the resistance of a three-dimensional wedge near the top of the rock, p_{us} , evaluated with the parameters at a confining stress representative of the wedge (which depends on the depth H below the top of the rock, measured along the pile axis), and the resistance to flow around the pile at depth:

$$p_{ud} = \left(\frac{\pi}{4} p_L + \frac{2}{3} \tau_{max} - p_a \right) D, \quad p_a = K_a \sigma'_v - 2 c' \sqrt{K_a} \geq 0 \quad (5.43)$$

where p_L is the major principal stress at failure for $\sigma_3 = \sigma'_v$ and τ_{max} the shear resistance of the pile–rock interface, which increases with the square root of σ_{ci} . The wedge resistance p_{us} follows the force equilibrium of the wedge given by Liang et al. (2009).

5.13 Calcareous rock and calcareous sand

5.13.1 Calcareous rock

Basis: Abbs (1983), for weak carbonate rock with a uniaxial compressive strength q_u of about 0.5 to 5 MPa, which fails in a brittle manner. The intact response is the static curve for stiff clay with free water (Section 5.3) with $c = q_u/2$: the ultimate resistance (5.6), ε_{50} from the strength (Section 5.16), the initial line $p = kxy$ up to $A_3 y_{50}$, and (5.7) and (5.8) up to the peak at $y_{pk} = 6A_3 y_{50}$. The initial modulus gradient k is 67 900, 135 700 and 271 400 kN/m³ for c below 48, 96 and 192 kPa, and 2835c

kN/m³ (c in kPa) for higher strengths. Beyond the peak the resistance falls linearly to the residual curve, which it reaches at $y = (1 + f)y_{pk}$; f is the displacement factor after peak (default 0.1). The residual curve is the cyclic sand curve of Reese, Cox and Koop (1974) (Section 5.6) with the residual friction angle φ_r , $K_0 = 0.4$ and the default k for φ_r . Carbonate materials commonly show residual friction angles of 24° to 33° after particle crushing (Abbs, 1983).

5.13.2 Calcareous sand

Basis: Guo and Lehane (2016), from lateral load tests on drilled and grouted piles in weak calcareous sandstone with cone resistances q_c of 20 to 50 MPa. The curve is bilinear:

$$p = \min \left(SF q_c y, 0.045 q_c b \right) \quad (5.44)$$

where SF is the stiffness factor entered with the material (default 12, the ratio E_s/q_c found at the test site), and $0.045q_c$ is the ultimate pressure measured on the 340 mm piles of that site. Guo and Lehane (2016) take the initial slope of the p - y curve as $0.8E_s$, about $9.6q_c$, and suggest $p_u \approx 0.06q_c b$ for design, noting that this may be conservative at large depths; their p - y data cover depths of $1.5b$ and more. Enter $SF = 9.6$ to use their initial slope. The curve does not depend on the effective stress and is the same for static and cyclic loading.

5.14 Other models

5.14.1 Piedmont residual soil

Basis: Simpson and Brown (2003). The secant modulus degrades logarithmically with the normalised deflection:

$$E_s = E_{si} \left(\frac{y}{b} \leq 0.001 \right), \quad E_s = E_{si} \left[1 + \lambda \ln \left(\frac{y/b}{0.001} \right) \right] \quad \left(0.001 < \frac{y}{b} \leq 0.0375 \right), \quad p = E_s b y \quad (5.45)$$

where λ is the (negative) degradation coefficient of the model; beyond $y/b = 0.0375$ the resistance is constant. The initial modulus E_{si} (force/length³) is entered directly or derived from a standard penetration, cone penetration, dilatometer or pressuremeter test with correlations of the type proposed by Simpson and Brown (2003).

5.14.2 Loess

Basis: Johnson et al. (2007), from load tests of drilled shafts in loess with cone penetration data. With the cone resistance q_c , reduced near the surface to $q_c(0.5 + 0.5z/2b)$ for $z < 2b$, where z is the distance from the pile head (a pile head above the ground therefore reduces the shallow reduction):

$$p_u = \frac{0.409 q_c b}{1 + 0.24 \log_{10} N}, \quad E_i = \frac{p_u}{y_{ref}}, \quad p = \frac{E_i y}{1 + y_h}, \quad y_h = \frac{y}{y_{ref}} \left(1 + 0.10 e^{-y/y_{ref}} \right) \quad (5.46)$$

with $y_{ref} = 2.97$ mm and N the number of cycles (1 for static loading, at most 10).

5.14.3 Elastic subgrade

The reaction is linear, $p = k_h b y$, with the modulus of subgrade reaction k_h (force/length³) constant within the layer. The model has no ultimate resistance.

5.14.4 User-defined curves

The curve is the polygon through the entered points, with the slope of the first segment from the origin and a constant reaction beyond the last point.

5.15 Layered soil

The ultimate resistance of the shallow wedge in a layer depends on the soil above it. Solpile applies the equivalent-depth method of Georgiadis (1983). For a layer whose top lies at depth H_i , an equivalent depth h_i' is found such that the ultimate resistance of the layer material, integrated from the ground surface to h_i' , equals the resistance actually provided by the overlying layers:

$$\int_0^{h_i'} p_{u,i}(x) dx = \sum_{j < i} \int_{\text{layer } j} p_{u,j}(x) dx \quad (5.47)$$

Within layer i the wedge terms of the p - y curve are then evaluated at the depth $x - H_i + h_i'$, while the effective stress remains that at the actual depth. Optionally the correction is omitted where a layer lies below a layer of the same p - y model. Calcareous rock takes part in the correction as a cemented c - φ soil with $c = q_u/2$ and φ equal to its residual friction angle; calcareous sand (CPT) keeps its actual depth.

5.16 Default parameters

When ε_{50} or the modulus gradient k is entered as zero, Solpile assigns a default value following the recommendations published with the model:

Table 5.2 — Basis of default parameters

Parameter	Models	Default depends on
ε_{50}	soft clay; stiff clays; cemented soils	undrained shear strength (consistency of the clay), after Matlock (1970) and Reese et al. (1975)
k , clay	stiff clay with free water; modified stiff clay	undrained shear strength and type of loading (static or cyclic), after Reese et al. (1975)
k , sand	sand (Reese); API sand	friction angle and position relative to the water table, after Reese et al. (1974) and API (2000)
k , cemented soils	silt (cemented soils)	cohesion, friction angle, position relative to the water table and type of loading

The default is a generic value for the soil type. For design, determine ε_{50} and k from site-specific tests where possible and enter them. The effect of a default on the curves can be inspected in the generated p - y curves at the output depths. The model check warns when a default k for sand is requested for a friction angle outside 29°–41°.

5.17 Moment due to side resistance

For large-diameter shafts the vertical shear on the shaft perimeter produces a resisting moment when the shaft rotates. With the ultimate unit side friction $f_s(x)$ entered by the user, Solpile applies a distributed moment

$$m(x) = f_s b^2 \min \left(1, 40 |S(x)| \right) \quad (5.48)$$

which is fully mobilised at a rotation of 0.025 rad. The distributed moment is introduced into the equilibrium equation as the equivalent lateral load $-dm/dx$ and updated in each iteration.

The sign of the distributed moment follows the sign of the entered side friction, not the local direction of rotation; only the uppermost segment follows the direction of the head deflection. Two consequences follow: where the rotation reverses (in the lower part of the pile) the side moment does not oppose it, and the response to a reversed lateral load is not the exact mirror of the response to the original load (differences of up to about 10 % in head deflection have been observed). If the loading direction matters, analyse both directions.

5.18 Lateral soil movement

When the free-field soil moves laterally by $y_s(x)$, the soil reaction depends on the relative displacement:

$$p = p_{curve}(y - y_s) \quad (5.49)$$

In the secant formulation the reaction is written as $p = -E_s(y - y_s)$, so that the soil movement contributes a load $E_s y_s$ to the right-hand side of (3.1). The following exceptions apply:

- in layers of massive rock, Piedmont residual soil and loess, the secant modulus is evaluated at the relative displacement, but no load is applied from the soil movement;
- in layers of strong rock the soil movement is ignored.

5.19 Ground slope and pile batter

A ground slope or a pile batter changes the size of the passive wedge in front of the pile. The effective angle θ combines the slope and batter angles. For the clay models the wedge resistance is multiplied by

$$f_{down} = \frac{1}{1 + \tan \theta'}, \quad f_{up} = \frac{1}{1 - \tan \theta} \quad (5.50)$$

for deflection down and up the slope respectively, while the flow-around resistance at depth is not modified. For the sand models the wedge equation is re-derived for the inclined surface. With $t = \tan \beta \tan \theta$ and the Coulomb coefficient $K_A = \cos \theta (\cos \theta - \sqrt{(\cos^2 \theta - \cos^2 \varphi')}) / (\cos \theta + \sqrt{(\cos^2 \theta - \cos^2 \varphi')})$, the terms of (5.21) are modified by the factors

$$D_1 = \frac{t}{t + 1}, \quad D_2 = 1 - D_1, \quad D_3 = \frac{t}{1 - t^2}, \quad D_4 = \frac{1 + t}{1 - t} \quad (5.51)$$

for the downhill direction (D_1, D_2) and the uphill direction (D_3, D_4). The resulting curves are asymmetric (Section 5.1).

5.20 p- and y-multipliers

A modified curve is obtained from

$$p'(y) = P_m p\left(\frac{y}{Y_m}\right) \quad (5.52)$$

with P_m and Y_m interpolated linearly in depth between the rows of the multiplier table, and equal to 1 above the first row and below the last row.

6 Section mechanics

6.1 Method

The bending response of nonlinear sections is obtained from a moment-curvature analysis by fibre integration. The concrete area is divided into thin strips parallel to the neutral axis; reinforcing bars, strands, casings and cores are represented at their actual positions. For a given curvature ϕ and neutral-axis depth c the strain varies linearly over the section,

$$\varepsilon(z) = \phi (c - z) \quad (6.1)$$

where z is measured from the compressed face. For each curvature the neutral axis is found such that the resultant normal force equals the axial thrust, and the moment and secant stiffness follow:

$$\sum_i \sigma(\varepsilon_i) A_i = P, \quad M = \sum_i \sigma(\varepsilon_i) A_i (z_g - z_i), \quad EI = \frac{M}{\phi} \quad (6.2)$$

with z_g the depth of the centroid. The curvature is increased stepwise until the extreme compressive strain reaches the end of the concrete stress-strain curve or the reinforcement reaches its limiting strain.

Where the computed moment decreases with increasing curvature, for example immediately after cracking, the moment is held at its previous maximum. The relation between moment and bending stiffness then remains single-valued, which the iteration of Section 3.3 requires.

6.2 Concrete

Unconfined concrete in compression follows a parabola up to the strain ε_0 at peak stress and a linear descending branch to the strain $\varepsilon_u = 0.0038$:

$$f_c = f'_c \left[\frac{2\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \quad (0 \leq \varepsilon \leq \varepsilon_0), \quad f_c = f'_c \left[1 - 0.15 \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right] \quad (\varepsilon_0 < \varepsilon \leq \varepsilon_u) \quad (6.3)$$

$$\varepsilon_0 = \frac{1.7 f'_c}{E_c}, \quad E_c = 4733 \sqrt{f'_c} \text{ (MPa)} \quad (6.4)$$

In tension the same parabola applies up to the rupture stress $f_r = 0.623 \sqrt{f'_c}$ MPa (0.332 $\sqrt{f'_c}$ MPa for prestressed sections); beyond cracking the tensile stress is zero. Values of E_c and f_r entered by the user replace the derived values. The laws are plotted in Figure 6.1.

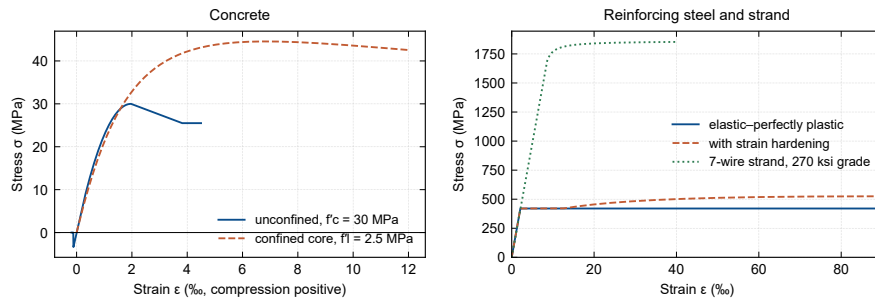


Figure 6.1 — Stress-strain laws as implemented (computed by Solpile)

6.3 Confined concrete

When confinement is selected, the concrete inside the transverse reinforcement follows the model of Mander, Priestley and Park (1988), while the cover follows the unconfined law. With the effective lateral confining stress f'_1 derived from the transverse reinforcement,

$$f'_{cc} = f'_c \left(2.254 \sqrt{1 + 7.94 \frac{f'_1}{f'_c}} - 2 \frac{f'_1}{f'_c} - 1.254 \right), \quad \varepsilon_{cc} = 0.002 \left[1 + 5 \left(\frac{f'_{cc}}{f'_c} - 1 \right) \right] \quad (6.5)$$

$$f_c = \frac{f'_{cc} \xi r}{r - 1 + \xi}, \quad \xi = \frac{\varepsilon}{\varepsilon_{cc}}, \quad r = \frac{E_c}{E_c - f'_{cc}/\varepsilon_{cc}} \quad (6.6)$$

The ultimate strain of the confined concrete, which ends the moment-curvature curve, is taken as (Priestley, Seible and Calvi, 1996)

$$\varepsilon_{cu} = 0.004 + \frac{1.4 \rho_s f_{yh} \varepsilon_{su}}{f'_{cc}} \quad (6.7)$$

with the volumetric ratio ρ_s and yield stress f_{yh} of the transverse reinforcement and $\varepsilon_{su} = 0.09$.

6.4 Reinforcing and structural steel

Steel is elastic-perfectly plastic with modulus E_s and yield stress f_y . When strain hardening is selected, the stress beyond the onset of hardening ε_{sh} is

$$f_s = f_y \left[\frac{m e + 2}{60 e + 2} + \frac{e(60 - m)}{2(30 q + 1)^2} \right] \leq f_u, \quad m = \frac{(f_u/f_y)(30 q + 1)^2 - 60 q - 1}{15 q^2} \quad (6.8)$$

with $e = |\varepsilon| - \varepsilon_{sh}$ and $q = \varepsilon_{su} - \varepsilon_{sh}$ (Park and Paulay, 1975). Steel pipes, H sections, casings and cores are integrated with the same law over their actual geometry. Steel C sections (with or without stiffening lips) about the strong axis, T sections about the weak axis, box sections and the AISC shape families (W, M, S and HP shapes, channels, tees, single and double angles, and hollow structural sections) are described by the width of steel across the bending axis at each distance from the extreme fibre, assembled from plates, rounded corners and rings. Unsymmetrical shapes bend about the geometric axis through their actual centroid. Dimensions of AISC shapes are taken from the AISC Shapes Database (AISC, 2013).

6.5 Prestressed sections

Seven-wire strands follow the stress–strain relations of the PCI Design Handbook (PCI, 2010): linear with modulus 196.5 GPa (28 500 ksi) up to the limit of proportionality, then

$$f_{ps} = f_{pu} - \frac{A}{\varepsilon_{ps} - B} \quad (6.9)$$

with $(\varepsilon_{lim}, A, B) = (0.0076, 0.04, 0.0064)$ for grade 250, $(0.0086, 0.04, 0.007)$ for grade 270, and $(0.0088846, 0.0835, 0.0071)$ for grade 300, stresses in ksi. Prestressing bars are elastic–plastic with modulus 200 GPa (29 000 ksi) and the strength of the selected bar (smooth bars 145 or 160 ksi; deformed bars 150, 157 or 160 ksi depending on the size), and carbon-fibre cables (CFCC) are linear-elastic up to rupture. The list of strand and bar sizes, with their areas, depends on the selected strand type. The effective prestress, the prestress force reduced by the fraction of losses, is introduced as an initial strain of the tendons.

6.6 Nominal moment capacity and interaction diagram

The nominal moment capacity M_n at a given axial thrust is the moment at which the extreme concrete compressive strain reaches 0.003, interpolated on the moment–curvature curve. Repeating the computation for a series of axial thrusts gives the interaction diagram. Axial thrusts outside the nominal axial capacity in tension or compression are rejected.

For a conventional analysis the demand interaction diagram adds, for each section, one point per load case: the axial thrust and the largest moment within the section.

6.7 Bending stiffness in the pile analysis

For each load case, the moment–curvature relation of every nonlinear section is computed at the axial thrust of the load case. During the iteration (Section 3.3) the bending stiffness of each node is taken from that relation at the current bending moment. Below the first computed point the uncracked stiffness is used. When the moment exceeds the largest moment of the relation, the stiffness of the last point is used and the warning CURVATURE-LIMIT is issued, because the section has then reached the end of its computed response.

Elastic sections have constant stiffness. For an elastic section with a specified moment capacity M_p , the response is elastic–perfectly plastic: $M = EI\varphi$ up to M_p and $M = M_p$ beyond. For a user-defined nonlinear section, the entered moment–curvature or stiffness–moment data are interpolated, including between the entered axial thrusts.

In a section with a specified moment capacity the capacity is not interpolated between the top and bottom values. The yield stress is taken from the capacity at the top of the section and applied to the dimensions at each depth: the section modulus of the interpolated diameter for circular; pipe and embedded-pole sections, and $M_p(z) = M_{p,top} b' h^2 / (b_{top} h_{top}^2)$ for rectangular sections, with the depth h interpolated over the section and $b' = b_{bottom} + (b_{bottom} - b_{top}) t$, where t is the relative position in the section. A section of constant dimensions therefore has the top capacity over its whole length. The node at the pile head uses the top capacity. General sections interpolate the capacity linearly; H and AISC shapes have a single capacity. The bottom capacity is kept in the file and drawn in the moment–curvature charts.

6.8 Section options

Equivalent diameter for p-y curves. When the option is selected, the width b used to generate the p-y curves of the nodes in the section is replaced by the entered equivalent diameter D_{eq} ; the bending stiffness, the moment–curvature relation and the capacities keep the actual dimensions. For tapered Elastic sections of rectangular, circular, pipe and embedded-pole shape, D_{eq} varies linearly from the top value to the bottom value (a bottom value of zero means the top value). The option is not used for user-defined nonlinear sections and for General Elastic sections. The value proposed when the option is selected is the diameter of the circle whose area equals that of the bounding rectangle of rectangular; square, H, AISC, C, T and box shapes, of the octagon of octagonal sections, and the outer diameter of round sections and pipes.

Equivalent elastoplastic moment–curvature (CALTRANS). Following the Caltrans Seismic Design Criteria (Caltrans, 2013), the computed curve (φ_k, M_k) is replaced by a bilinear one. The anchor point a is the first yield of the reinforcement or of the steel tube (the first cracking of the concrete for prestressed sections), and the elastic stiffness is $El_{el} = M_a/\varphi_a$. The plastic moment M_p makes the areas under the two curves equal beyond the anchor,

$$\sum_{k \geq a+2} (\varphi_k - \varphi_{k-1}) \frac{\min(EI_{el} \varphi_k, M_p) - M_k}{2} = 0 \quad (6.10)$$

and is found by bisection to a relative tolerance of 10^{-4} of the area under the computed curve. The idealised curve is $M = El_{el} \varphi$ up to the idealised yield curvature $\varphi_y' = M_p/El_{el}$ and $M = M_p$ up to the last computed curvature φ_u ; the plastic curvature capacity is $\varphi_u - \varphi_y'$.

Adjusted softening. With the option on (the default), each moment of the computed curve is replaced by the largest moment reached at a smaller curvature, so that the curve, and with it the bending stiffness used in the pile analysis, does not follow a falling branch after the peak. With the option off the computed moments are used as they are.

7 Special analyses

7.1 Pile-head stiffness

The lateral stiffness terms of the pile head are computed at n load levels. The levels are fractions u_i of the maximum value, either arithmetic or logarithmic:

$$u_i = \frac{i}{n} \quad (\text{arithmetic}), \quad u_i = \frac{1}{n} + \left(1 - \frac{1}{n}\right) \frac{\ln i}{\ln n} \quad (\text{logarithmic}), \quad i = 1 \dots n \quad (7.1)$$

At each level two analyses are made: with head condition type 2, shear $u_i V$ and zero rotation, giving $K_{22} = V/y_t$ and $K_{32} = M_t/y_t$; and with head condition type 4, zero deflection and moment $u_i M$, giving $K_{23} = V_t/\theta_t$ and $K_{33} = M/\theta_t$. V and M are those of load case 1, or derived from a maximum head deflection and rotation. Nonlinear sections use the moment–curvature relation at the thrust of load case 1.

7.2 Pushover analysis

The head deflection is increased in steps from a minimum to a maximum value (arithmetic, logarithmic or user-defined spacing). With logarithmic spacing the k -th of n deflections is $y_k = y_{\min} + (y_{\max} - y_{\min}) \ln k / \ln n$; a zero minimum is replaced by $y_{\max}/n$. For a free (pinned) head each step uses head condition type 4 with zero moment; for a fixed head, type 5 with zero slope. For each step the head shear, the maximum moment and its depth are recorded. Zones in which the moment reaches the plastic plateau of the moment–curvature relation are reported as plastic hinges; the analysis can be stopped at the formation of the second hinge.

Alternatively the pushover is based on load case 1: the loads of load case 1 are multiplied by a series of factors from a minimum to a maximum value (arithmetic, logarithmic or user-defined spacing), and each factor is analysed with the head condition of load case 1. A factor of 1 reproduces load case 1.

7.3 Buckling analysis

A lateral head load is held constant while the axial thrust is increased in n steps from $P_{\min}$ to $P_{\max}$. The head deflection increases without bound as the thrust approaches the buckling load; the sweep stops at the first step that does not converge. The buckling load is estimated by fitting a hyperbola to the computed points. With (P_1, y_1) the first point,

$$P = P_1 + \frac{y - y_1}{b + a(y - y_1)} \quad (7.2)$$

The coefficients follow from a least-squares fit of the straight line

$$\frac{y - y_1}{P - P_1} = b + a(y - y_1) \quad (7.3)$$

and the estimated buckling load is the asymptote $P_{cr} = P_1 + 1/a$. The estimate is reliable when the sweep reaches the strongly nonlinear part of the curve.

7.4 Head deflection versus pile length

The analysis is repeated for pile lengths from a maximum value down to a minimum value, with the soil profile and the head load unchanged, and the head deflection, maximum moment and maximum shear are recorded for each length. For long piles the head deflection is independent of the length; below a critical length it increases rapidly. The head load is that of the sweep (shear with moment, slope or rotational stiffness, or a specified head displacement with moment or slope, analysed as head condition types 1–5) or, when it is zero, load case 1. With the number of lengths N given, the lengths are $L_{\max} - k\Delta L$ for $k = 0 \dots N-2$ followed by $L_{\min}$ ($N = 1$: $L_{\min}$ only); with the increment given, $N = [(L_{\max} - L_{\min})/\Delta L] + 1$ lengths $L_{\max} - k\Delta L$, $k = 0 \dots N-1$, are analysed. The increment is used as entered (a zero increment repeats the maximum length) and lengths of zero or less are skipped. A length without a solution is reported and the sweep continues; without entered limits, the default sweep from the full length down to 20 % of it stops at the first length without a solution, which indicates that the pile has become too short to carry the load.

7.5 Shear demand along the pile

Basis: Reese, Wang and Fouse (1992), for piles used to stabilise a slope. For each assumed depth L_s of the sliding surface below the pile head, the soil above L_s is displaced by a lateral soil movement profile and the soil below L_s does not move; the movement acts through the p-y springs as described in Section 5.18. The profile is either linear, from the movement at the ground surface to that at the pile tip, or a table of depths and movements; it ends at L_s with a linearly interpolated value. The pile head carries the load entered for this analysis. For each L_s Solpile reports the head deflection, rotation, shear and moment, the moment and shear at L_s , and the maximum moment. The depths L_s are evenly spaced between a minimum and a maximum, stepped by an increment, or entered as a list.

7.6 LRFD check

For each load combination the factored head shear, moment and axial force are formed from the unfactored loads by (7.4), and the pile is analysed with head condition type 1.

$$(V, M, P)_i = \sum_c \gamma_{i,c} \sum_{j \in c} (V_j, M_j, P_j) \quad (7.4)$$

For every section the largest developed moment and shear within its length are compared with the factored capacities $\phi_t M_n$ and $\phi_v V_n$, where M_n is the nominal moment capacity at the factored axial force and V_n the shear capacity entered for the section. A combination passes when both ratios are not greater than 1.

When **Estimate Resistance Factors and Shear Capacity** is selected for a round or rectangular reinforced-concrete section without confined concrete, the resistance factors follow ACI 318 ($\phi_v = 0.75$) and $V_n = V_c + V_s$ with $V_c = 2\sqrt{f'_c} b d$ (psi; $0.166\sqrt{f'_c}$ MPa), $\sqrt{f'_c}$ not taken greater than 100 psi, and $V_s = A_v f_{yt} d/s \leq 4V_c$, where A_v is the area of two legs of the transverse bars and s their spacing. For round sections $d = 0.8D$ (V_s : 0.85D with a manual bar layout) and $b = D$; for rectangular sections d is the depth to the centre of the extreme tension bar (0.85h with a manual bar layout).

8 Verification

8.1 Approach

The implementation is verified at three levels: against closed-form solutions of the governing equation, by the convergence of the discretisation, and by checks of equilibrium and of section properties that do not depend on the p-y models. The p-y models themselves are empirical; their validity rests on the field tests reported in the publications cited in Chapter 5. All results in this chapter were computed with Solpile 1.0.0 and are reproducible with the scripts supplied with the documentation.

8.2 Elastic pile in uniform elastic soil

For a semi-infinite pile of constant bending stiffness in soil with constant secant modulus $E_s = k_h b$ and constant axial thrust P , the governing equation (2.3) reduces to

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + E_s y = 0 \quad (8.1)$$

whose decaying solution is $y = \text{Re}[A e^{rx}]$, where r is the root of $EI r^4 + P r^2 + E_s = 0$ with negative real part and positive imaginary part, and the complex constant A follows from the two head conditions. For $P = 0$ this is the classical solution of Hetényi (1946), with the characteristic parameter

$$\lambda = \left(\frac{E_s}{4EI} \right)^{1/4}, \quad y_0 = \frac{2V\lambda}{E_s} \quad (\text{free head, shear } V) \quad (8.2)$$

The test pile is 30 m long ($\lambda L = 10.6$, a long pile), with $D = 0.6$ m, $E = 30$ GPa, $EI = 190852$ kN·m², and an elastic subgrade $k_h = 20\,000$ kN/m³, so that $E_s = 12000$ kN/m² and $\lambda = 0.3541$ m⁻¹. The Solpile model uses 400 increments.

Table 8.1 — Comparison with the closed-form solution

Case	y_0 exact (mm)	y_0 Solpile (mm)	Diff. (%)	$M_{\max}$ exact (kN·m)	$M_{\max}$ Solpile (kN·m)	Diff. (%)
A — free head, $V = 100$ kN	5.9014	5.9004	-0.018	91.05	91.01	-0.040
B — free head, $M = 100$ kN·m	2.0896	2.0889	-0.035	100.00	100.00	0.000
C — fixed head, $V = 100$ kN	2.9507	2.9512	0.018	141.21	141.18	-0.018
D — as A, with $P = 6000$ kN	6.5324	6.5311	-0.020	106.03	106.00	-0.031

The differences are below 0.04 % for deflection and moment in all four cases, including the second-order effect of the axial thrust in case D, which increases the head deflection by $0.0065324/0.0059014 - 1$, about 11 %. The maximum moment of Solpile is the largest nodal value, so it includes the effect of the node spacing on the location of the maximum. The profiles of case A are compared in Figure 8.1.

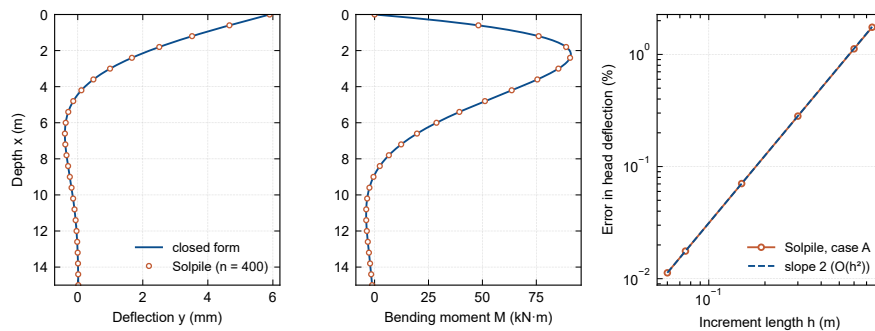


Figure 8.1 — Case A: deflection and moment profiles, and convergence of the head deflection with the increment length

8.3 Convergence of the discretisation

Case A was repeated with 40 to 500 increments. The error in head deflection decreases with the square of the increment length, as expected for the central-difference scheme (Figure 8.1, right):

Table 8.2 — Error of the head deflection, case A

Increments N	h (m)	y_0 (mm)	Error (%)
40	0.750	5.7983	1.747
50	0.600	5.8352	1.122
100	0.300	5.8848	0.282
200	0.150	5.8972	0.070
400	0.075	5.9004	0.018
500	0.060	5.9007	0.011

In this case the relative error of the head deflection is very close to $(\lambda h)^2/4$, which can serve as an estimate for long piles in uniform soil. With the default of 100 increments it is a few tenths of a per cent for typical piles.

8.4 Equilibrium of nonlinear solutions

For a nonlinear analysis no closed-form solution exists, but the converged solution must satisfy global equilibrium. For the layered-soil problem of Example 1 of the User Manual (steel pipe pile, three soil layers, axial thrust 500 kN), horizontal equilibrium $V_0 + \int p \, dx = 0$ and moment equilibrium including the P-Δ moment, $\int p \, x \, dx - M_0 - P(y_0 - y_L) = 0$, were evaluated from the computed profiles:

Table 8.3 — Residuals of global equilibrium, Example 1

Load case	Force residual (kN)	Moment residual (kN·m)	P-Δ moment (kN·m)
Free head, 100 kN	1.7e-10	-4.7e-10	7.818
Free head, 200 kN	-4.3e-10	-3.4e-10	21.220
Fixed head, 200 kN	-6.3e-10	1.5e-10	5.376

The residuals are at the level of round-off. They confirm that the soil reactions, the internal forces and the second-order moment are mutually consistent.

8.5 Section properties

For the steel pipe of Example 1 ($D = 610$ mm, $t = 12.7$ mm, $f_y = 355$ MPa, $E = 200$ GPa) the fibre analysis at zero thrust was compared with the closed-form properties of the annulus:

Table 8.4 — Steel pipe: fibre analysis against closed form

Quantity	Closed form	Solpile	Diff. (%)
Elastic bending stiffness EI (kN·m ²)	212651.0	212651.2	0.0001
Plastic moment $f_y Z$ (kN·m) / largest computed moment	1608.7	1597.8	-0.68

The elastic stiffness agrees to the precision of the integration. The largest computed moment approaches the fully plastic moment from below; the remaining difference reflects the curvature at which the computed relation ends, at which a thin core of the wall is still elastic.

8.6 Regression testing

Each release of Solpile is checked against an internal suite of automated tests that covers every p-y model, every section type and every special analysis, including the cases of this chapter. A release is issued only when the suite passes.

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